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NUMBER SYSTEMS OF THE NORTH AMERICAN INDIANS.

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II. SYSTEMS OF NUMERATION.

Counting cannot be carried far by the use of successive unrelated terms or symbols for each number. For higher terms some system of compounding is necessary, the usual method being by reference to some stopping point or *base* which is thought of as a new starting point. The choice of this base is of fundamental importance in the development of a true *number system* as distinguished from a mere *series of numbers*. The choice of this base seems usually to be related to primitive finger counting. One hand, two hands, or all the fingers and toes, are the three most natural stopping points, leading respectively to quinary, decimal, and vigesimal systems as illustrated below. There is much variation in the choice of these bases and in the way in which they are built up into systems for actual use after the base has been established. Thus a pure quinary system is rare, usually merging into a decimal or vigesimal one for the higher numbers. Accordingly in the list given below of the number of instances of each system found, quinary and quinary-decimal systems have been classed together, and similarly vigesimal and quinary-vigesimal.¹ The numbers found are: decimal 146, quinary and quinary-decimal 106, vigesimal and quinary-vigesimal 35, quaternary 15, ternary 3, octonary 1 (binary 81). The binary instances given in parentheses refer simply to languages in which the duplicative principle (already discussed) occurs. These may be thought of as traces of a binary system. But there are no binary systems in the true sense of the word.

1. **Decimal Systems.** Many of the American Indian systems have decimal scales as regular and as complete as our own, extending to quite high limits.

¹ Probably several systems have been wrongly classified, due to inadequate data. Many authorities give the numerals from 1 to 10 and then for 100, or perhaps by tens to 100. The greatest error is probably in the classification of too many languages as decimal. This has been done in some cases from comparison with cognate languages, where the direct data were insufficient. Some have been called quinary-decimal, where further data would probably show they were actually quinary-vigesimal. These variations in detail will probably not seriously affect the general conclusions stated.

Many others, while predominatingly decimal, are combined with other systems, decimal-quinary and decimal-vigesimal being most frequent. The decimal system appears exclusively (i. e., no elements of other systems have been found) in only eleven families, all of which are small ones totaling only 19 languages of those examined. Thus in only a comparatively few cases do the languages of a single family use the decimal system consistently.

As illustrations we find "one hundred" expressed by a unique word or by such forms as "completed," "stock of tens," " 10×10 "; "thousand" by " $10 \times 10 \times 10$," " 10×100 ," "big hundred," "old man hundred," "large stock of tens"; "million" by " $1,000 \times 1,000$," "big thousand," "too many to count." While there are many variations, intermediate numbers as a rule are formed as in English.

2. Quinary and Quinary-decimal Systems. As already mentioned a pure quinary system is very rare, if indeed one exists. The pure form would require only five elements and is of course 1, 2, 3, 4, 5, $5 + 1$, $5 + 2$, $5 + 3$, $5 + 4$, 2×5 , and so on to 3×5 , 4×5 and 5×5 as a new primary base. There are various approximations to such a system. Even 10 as " 2×5 " is uncommon although several instances of this have already been given. We have as variations for the numbers from 6 to 9, $6 = X + 1$ ("X" standing for some descriptive, non-numerical word), $7 = X + 2$, etc., arising from such forms of thought as "again 1," "second 1," "1 more," "on the other, 1," the numerals of the second quinate repeating without the use of the expressed base *five*. One or more of these forms may be entirely lacking, 8 being expressed " 2×4 " and similarly. But only in case at least two of the numbers between 5 and 10 show such a formation have we classed it as quinary. *Nine* is a frequent variation, since it is so often expressed by the subtractive principle. With these explanations we may state that quinary systems occur in about one third of the languages examined, appearing most frequently in the region around the Gulf of Mexico and least in the languages along the Northwest Coast.

Space will not permit illustrative systems to be given in full. Luiseno is purely quinary to ten,¹ Gallinomero has only a few variant forms until it reaches forty, where it changes to decimal,² while most of the quinary systems are of the general type, 1, 2, 3, 4, 5, $X + 1$, $X + 2$, $X + 3$, $X + 4$ and a new word for ten, e. g. Delaware³ or Shawnee.⁴

3. Vigesimal and Quinary-vigesimal Systems. Vigesimal systems more or less complete occur in about one tenth of the languages examined. With the exception of the Caddoan family in the middle west they all appear along the Pacific Coast or in the far north. The obvious explanation of this system is in the digital origin of counting. The well known ethnologist Gatschet says that

¹ Dixon & Kroeber: "Numeral systems of languages of California," in *Amer. Anthropologist*, n.s., Vol. 9, p. 681.

² Dixon & Kroeber: *op. cit.*, p. 676.

³ Zeisberger, D., "Grammar of the Delaware," in *Amer. Philosophical Soc. Trans.*, n.s., Vol. 3, p. 65.

⁴ Johnson, J., "Present State of Indian Tribes Inhabiting Ohio," in *Amer. Antiquarian Soc. Proc.*, Vol. 1, p. 269.

"tribes living in tropical and hot climates mostly possess the vigesimal system of notation, which is rather infrequent among the Indians of the United States."¹ He finds the explanation in the fact that they live barefoot as contrasted with the moccasined northern Indians. But this "barefoot" explanation, also given by other writers (especially to account for the unusually well developed vigesimal systems of the Mexicans and Aztecs),² rather breaks down in the case of the Eskimo and tribes of the north Pacific Coast where the climate is scarcely adapted to the barefoot stage. And yet among the Eskimo the vigesimal system has found its fullest development north of Mexico. The use of fingers and toes for the development of a vigesimal system seems to be independent of climate.

Twenty is the primary basis of the vigesimal system and is usually expressed as "man," "Indian," "man completed"; the multiples of twenty being expressed as "two men," "three Indians," etc. Pawnee carries this to 1,000 which is "50 persons."³ The vigesimal system usually occurs in connection with some other system. Three types of combination may be noted.

(a) *Quinary-vigesimal*. This is most frequent. The Greenland Eskimo says "other hand two" for 7, "first foot two" for 12, "other foot two" for 17 and similar combinations to 20, "man ended." The Unalit is also quinary to twenty which is "man completed." But 40 is "two sets of animals' paws," 60 "three sets of animals' paws" and so on regularly to 400 where there is an interesting change in the formation of this primary base (20×20) from animals back to man, for 400 is "20 sets of *man's* paws."⁴

(b) *Decimal-vigesimal*. Systems in which no quinary elements are found are comparatively rare. Wintun is alternately decimal and vigesimal, 20, 40, 60, being "one Indian," "two Indian," "three Indian," while 30, 50 are " 3×10 ," " 5×10 ."⁵ Others are similar.

(c) *Quinary-decimal-vigesimal*. Several systems show a combination of the three digital bases in their formation. Kopiagmiut is quinary to ten, decimal for the formation of the odd tens and vigesimal for the even ones.⁶ Amador is purely decimal to ten, quinary from ten to twenty, and then vigesimal, the odd tens being formed by addition of ten to the preceding even ones, e. g., $50 = 40 + 10$.⁷ Haida is quinary-decimal and quinary-vigesimal alternately to a hundred, then pure vigesimal, 400 being $20 \times 20 \times 1$ and 800 being $20 \times 20 \times 2$.⁸

4. Quaternary Systems. Fairly well defined quaternary systems reaching

¹ Gatschet, A. S.: "Indian Numeral Adjectives," in *Amer. Antiquarian*, Vol. 2, p. 210.

² The best developed vigesimal system on the American continent is probably the Maya of Mexico. This is given and discussed in Conant's *Number Concept* and in much fuller form by Thomas in the 19th Rpt. Bur. of Eth., p. 853.

³ Dunbar, J. B.: *The Pawnee Language*, 1893. (No place of publication given. Pamphlet, Copy in Newberry Library, Chicago.)

⁴ Nelson, E. W.: "Eskimo about Bering Strait," in 18th Ann. Rpt. Bur. of Eth., 1896-7, p. 238.

⁵ Dixon & Kroeber: *op. cit.*, p. 675.

⁶ Petitot, P.: *Vocabulaire Francaise Esquimau*, Paris, 1876.

⁷ Dixon & Kroeber: *op. cit.*, p. 680.

⁸ Harrison, C.: "Haida Grammar," in *Royal Soc. of Canada Proc. and Trans.*, Vol. 1, 2d Ser., Sect. 2, p. 123.

to eight may be found among the Montagnais of the far north, the Foxes of Wisconsin, the Iowas and Missouris of the Plains—but they find their best and fullest development into true systems in various California tribes. Usually they are mixed with other systems, but one or two cases are found of practically pure quaternary systems.

It is not easy to account for the origin of such a system. Two possibilities may be suggested. (a) *Digital*. Perhaps a few tribes, for reasons best known to themselves, did not use their thumbs in counting. This is possible but there is little if any linguistic or observational evidence to support it. Most frequently these systems show the words "stick," "middle," "body." "Body" might be considered digital. (b) *Sacred Number Theory*. Four was the sacred number of many widely separated tribes of Indians. The four cardinal points of the compass and the four seasons were recognized by the Indians. Among several tribes the literal meaning of "four" is "complete," "right," "perfect." However the systems of these tribes are not quaternary ones. A chart for the sacred rites of the Ojibwas shows four degrees of initiation for medicine men. The swastika, with its four arms, originated with the Indians of the Southwest and was much used in basketry. There is a widespread "four worlds" of Indian mythology. In the various languages spoken on Puget Sound the same word for four, with minor variations, is seen most frequently, and it is the only number word common to about a dozen of these languages which have been most carefully studied by a missionary among them. All these data are suggestive of the origin of a quaternary system, although the fact remains that most of the instances given above are from tribes which do not actually possess such systems.

Santa Barbara as far as sixteen is as follows: 1, 2, 3, 4, $X + 1$, $X + 2$, $X + 3$, 8, 9, $X + 2$, 11, 3×4 , $X + 1$, $X + 2$, $X + 3$, 16. For 20 and above it is decimal, probably due to contact with civilization. San Luis Obispo has $X + 2$, $X + 3$, for 6, 7; $8 + 1$ for 9; $12 + 1$, $12 + 2$, $12 + 3$ for 13, 14, 15.¹ Numerous other languages have many quaternary forms.

5. Ternary System. A ternary system is much rarer than a quaternary and nowhere occurs in a pure form. Two well-developed ones are given below. It is even more difficult than in the case of the quaternary system to account for its origin. It is possible that to some primitive minds it seemed natural to count one, two, three, and then by groups of threes. In some languages there is a similarity between the words for "this," "that," "that (remote)" or "here," "there," "yonder" and the first three numerals. A number of instances of 6 expressed as 2×3 have already been given² and some writers have mentioned them as examples of a ternary system. But for reasons already explained they are better considered as formed by the Duplicative principle, unless occurring in connection with nine or twelve similarly formed. The Cuchan numerals for 3, 6, 9, are ha-mook, hum-hook, hum-ha-mook.³ In San Antonio 4 is related to 1, 6 is derived from 3, and 12 is directly 4×3 , 15 is 5×3 , and 13 is $12 + 1$.⁴

¹ Dixon & Kroeber: *op. cit.*, p. 682.

² See "Duplicative Principle" already discussed.

³ Trumbull: *op. cit.*, p. 41.

⁴ Dixon & Kroeber: *op. cit.*, pp. 683, 690.

But Coahuiltecan of Texas is the most interesting example found. It has binary, ternary, quaternary, quinary, decimal and vigesimal features,¹ but seems to be prevailingly ternary. Its system as far as 50 is as follows:

1	4	7 = 4 + 3	10 = 5 × 2
2	5	8 = 4 × 2	11 = 10 + 1
3 = 2 + 1	6 = 3 × 2	9 = 4 + 5	12 = 4 × 3
13 = 12 + 1	16 = 15 + 1	19 = 18 + 1	30 = 20 + 10
14 = 12 + 2	17 = 15 + 2	20	40 = 20 × 2
15 = 5 × 3	18 = 6 × 3?		50 = 40 + 10

6. Octonary System. A single system with a base eight is known, the Yuki of California. This interesting system in translation is:²

1	9 = beyond-1-hang	17 = 1-middle-project
2	10 = beyond-2-body	18 = 2-middle-project
3	11 = -3-body	19 = 3-middle-project
4	12 = -4-body	20 = 4-middle-project
5	13 = -5-body	
6	14 = -6-body	
7	15 = -7-body	
8	16 = middle-none	

7. Traces of Other Systems. Only traces of the use of other bases are found.

(a) *Binary.* If we include the large number of examples of the duplicative principle already discussed, we have many instances of binary elements. But these are only in the multiplicative principle and refer simply to doubling. The only place where we have found the additive principle used is in the Coahuiltecan (just given) where $3 = 2 + 1$. We may also notice Chutsinni: 2, *stunga*; 4, *stung-sung*; 8, *stun-sunga*. Yokaia: 2, *ko*; 4, *duo-ko*; 8, *ko-ko-dol*. (b) *Sexanary.* Aside from the instances of twelve expressed as 2×6 mentioned under the duplicative principle, we have Wimunche Ute: $7 = 6 + 1$, Rumsen: $7 = X + 6$, $8 = 2 + 6$. (c) *Base of Nine.* Trinity: $10 = 9 + 1$, $11 = 9 + 2$. (d) *Base of Forty.* Chwachamaju: 40, *ku-hai*, "1-stick"; 80, *ko-hai*, "2-stick." (e) *Base of Sixty.* Achomawi: $70 = 60 + 10$, $80 = 60 + 20$.

8. Conclusions. The most striking feature of the systems which have been studied is their diversity, even in languages of the same family, and much more marked when the country as a whole is considered. For instance in the closely related languages of the Yukian family in California, although the numerals from one to four are quite similar, yet two of the systems are quinary-decimal, a third is quinary-vigesimal, while the fourth is octonary; or in the Pujunan family in which one system is decimal, eight quinary-decimal and two quinary-vigesimal.

How many unique abstract number words are necessary for building up a number system? We recall that in English ten are used and that all up to one hundred are but combinations of these ten. Of course two are sufficient—in a binary system. Many of the Eskimo tribes manage quite well with five. The Luiseno of California has but five abstract number words, but it has higher units

¹ Gallatin, A.: *Trans. Amer. Ethnological Soc.*, Vol. 1, N. Y., 1845, p. 1.

² Dixon & Kroeber: *op. cit.*, pp. 677, 685. This system is given by these authors as quaternary, but they have acknowledged that this is an error, and that it should be classed as octonary.

which are chiefly descriptive phrases indicating various combinations of hands and feet. Many languages which have a decimal system get along easily without the full quota of ten as used in English. This is accomplished by the use of such combinations as $8 = 2 \times 4$, $6 = 2 \times 3$, 7, 8, 9 as 3, 2, 1 respectively subtracted from 10, and others which have already been given. The Coahuiltecan (given under the ternary system) forms all the numbers up to twenty with only four unique words, those for 1, 2, 4, 5—a very remarkable instance.

III. MISCELLANEOUS POINTS.

1. **Limits in Use.** The numerals in some languages are given as high as a million and in many others to a thousand or more. We are naturally led to inquire whether such high numbers were actually in use by primitive people. The evidence found on this point cannot be here given in detail. Only a few conclusions may be stated. The Eskimo seem to be poorest in ability to count, being low in comparison with the tribes of the United States. Most of them in ordinary conversation do not use above five or six, referring to higher numbers as “many,” but the more intelligent of them can count to 400. The Indians of the eastern United States who stand comparatively high intellectually, could use their numerals to 10,000 and their systems were such as to admit of indefinite expansion, one billion for instance being expressed as $1,000 \times 1,000 \times 1,000$. The Crees could count correctly as far as 1,000. The Winnebagoes are said to use their numerals as high as one million. Indefinite and countless numbers they represent by the terms “leaves on the trees,” “stars of the heavens,” “blades of grass on the prairie,” “sand on the lake shore.” The Crows do not count above a thousand, as they say honest people have no use for higher numerals! The Apaches cannot use numbers beyond 100,000. Any of the California tribes of which positive statements can be made can count into the hundreds. As a definite upper limit to counting ability we find that the Tuolomne are credited with the ability “to count with great rapidity almost to infinity”¹

Speaking in general terms, we may say that the rather highly developed Siouan tribes of the Plains, the Iroquoian and Algonquian families of the east, and the Muskogean tribes of the South—all rather high in the scale of civilization—could count intelligently at least into hundreds of thousands and had words for even higher numbers; that most of the other Indians of the country had little actual knowledge of forms for numbers higher than thousands; while the Eskimo of the far north were limited to hundreds and in many cases to twenty or even ten. It is probable that before coming in contact with European civilization the Indians had little occasion to use numbers beyond a thousand. But the systems of many of them were such as to admit of indefinite and easy extension when needed.

2. **Numeral Classifiers.** Some tribes use different sets of numerals for counting different classes of objects. The Tsimshian language has quite distinct forms for counting *men* and for counting *things*. More frequently the number stem is

¹ Johnson, A.: In Schoolcraft's *Indian Tribes* (*op. cit.*), Vol. IV, p. 406.

modified by a prefix or suffix which is in the nature of a classifier to denote the class of objects counted. The Haida has no less than 15 of these classifiers. Others seem to have an even larger number.¹ This usage is found very generally among the languages of the north Pacific coast and but rarely in other parts of the country. The Tsimsian mentioned above has different forms for abstract counting, for counting flat objects or animals, for round objects or time, for men, for long objects, for canoes, for measures.

3. **Verbal Nature.** In a few languages the numerals are true verbs instead of adjectives, and as such are conjugated through all the variations of mood, tense, person and number. As far as found this peculiarity is limited to three languages, Cree, Crow, Micmac.

4. **Derivative Numerals.** The formation of ordinals, adverbials and distributives from the cardinal numerals is more properly a grammatical than a numerical process and as such need receive only slight notice here. (a) *Ordinals* are found most frequently, usually being formed from the cardinals by a suffix or other terminal modification, occasionally by a prefix. In the Creek an unusual method is found, the ordinal being formed from the cardinal in the same way that the superlative of the adjective is formed from the comparative. (b) *Distributives* are also frequently found. They are formed from the cardinals by prefixes, suffixes, or reduplication. (c) *Adverbials*, as far as noted, are formed by suffixes.

5. **Arithmetical Operations.** We shall close our discussion of the varied, interesting, and intricate number systems of the North American Indians with a reference to their ability to perform arithmetical operations. In our study of the principles of formation of number words we found an extensive use of addition and multiplication, a less use of subtraction, and very slight use of division in the formation of number systems. But aside from these instances (all operations on only the *bases* of the systems) the calculative ability of the American Indians was very slight and of the most elementary sort. Addition, subtraction or multiplication was accomplished only with the aid of the fingers, sticks, pebbles or other convenient counters. It is probable that the native Indian mind had practically no idea of mental arithmetic, being unable to multiply or divide numbers mentally, or even to add or subtract any except the smallest. His need for such operations was probably as slight as his knowledge.

THE CURVE OF LIGHT ON A CORRUGATED DOME.

By WM. H. ROEVER, Washington University.

The Phenomenon. On a bright day a person situated at a high point, within a distance of three or four miles of the new Roman Catholic Cathedral of St. Louis, will observe a well-defined curve of light on the dome of the cathedral. This curve, which is closed, passes through the highest point of the dome and

¹ The Maya of Mexico has at least 75 such classifiers. See article by Mrs. Z. Nuttall in *Amer. Anthropologist*, N.S., Vol. 5, p. 667.